

## Semiconductor Materials: Physics and Technology

Resistivity:

$$\rho = \frac{1}{\sigma}, \quad R = \rho \cdot \frac{L}{Wh}$$

No. of occupied energy levels:

$$n = \int_{E_c}^{E_{top}} N(E) \cdot F(E) dE$$

$$p = \int_{-\infty}^{E_v} N(E) \cdot (1 - F(E)) dE$$

$$F(E) = \frac{1}{1 + e^{\frac{(E - E_F)}{kT}}}$$

Non degenerate s.c. carrier densities:

$$n = N_c e^{\left(-\frac{E_c - E_F}{kT}\right)}$$

$$p = N_v e^{\left(-\frac{E_F - E_v}{kT}\right)}$$

Intrinsic s.c. carrier density:

$$n = p = n_i(T), \quad n \cdot p = n_i^2$$

$$n_i = \sqrt{N_c N_v} e^{\left(\frac{-E_g}{2kT}\right)}$$

$$n_i^2 = 15,05 \cdot 10^{32} \cdot T^3 \cdot e^{(-14000/T)}$$

$$N_{c,v} \equiv 2 \left( \frac{2\pi m_{n,p}^0 kT}{h^2} \right)^{\frac{3}{2}}, \quad h = 2\pi \hbar$$

$$n = N_d, \quad p = \frac{n_i^2}{N_d}; \quad N_d \gg N_a, \quad N_d \gg n_i \quad n\text{-type}$$

$$p = N_a, \quad n = \frac{n_i^2}{N_a}; \quad N_a \gg N_d, \quad N_a \gg n_i \quad p\text{-type}$$

Neutralitätsbedingung im dot. s.c.:

$$n - p = N_d \quad n\text{-type}$$

$$p - n = N_a \quad p\text{-type}$$

Carrier densities in a doped s.c.:

$$n = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$p = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Fermilevel of an intrinsic s.c.:

$$E_i = \frac{E_c - E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_v}{N_c}\right) = \frac{E_c - E_v}{2} + \frac{3}{4} kT \ln\left(\frac{m_p^0}{m_n^0}\right)$$

Fermilevel in a doped s.c.:

$$E_F - E_i = kT \ln\left(\frac{n}{n_i}\right) = -kT \ln\left(\frac{p}{n_i}\right)$$

Drift velocity:

$$\vec{v}_d = \mu \vec{E}$$

Drift current equations:

$$J = \frac{I}{A}$$

$$J_{n,drift} = -qn\vec{v}_d = qn\mu_n\vec{E}$$

$$J_{p,drift} = qp\vec{v}_d = qp\mu_p\vec{E}$$

Einstein's relations:

$$D_n = \left(\frac{kT}{q}\right)\mu_n, \quad D_p = \left(\frac{kT}{q}\right)\mu_p$$

Resistivity of a s.c.:

$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

Conductivity of a s.c.:

$$\sigma = \sigma_n + \sigma_p = q(n\mu_n + p\mu_p)$$

Diffusion lengths:

$$L_n = \sqrt{D_n \tau_n}, \quad L_p = \sqrt{D_p \tau_p}$$

Diffusion currents:

$$J_{n,diff} = qD_n \frac{dn}{dx}, \quad J_{p,diff} = -qD_p \frac{dp}{dx}$$

Current equations:

$$J_n = J_{n,drift} + J_{n,diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_{p,drift} + J_{p,diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

Continuity equations:

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - R_n + G_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - R_p + G_p$$

## The pn-Junction

For an abrupt junction:

Build in Potential:

$$\phi_j = V_{th} \ln\left(\frac{N_a N_d}{n_i^2}\right), \quad V_{th} = \frac{kT}{q}$$

Field Distribution:

$$E(x) = \frac{1}{\epsilon_0 \epsilon_{si}} \int \rho(x) dx$$

$$E(x) = -\frac{qN_a}{\epsilon_0 \epsilon_{si}} (x_p + x), \quad -x_p \leq x \leq 0$$

$$E(x) = -\frac{qN_d}{\epsilon_0 \epsilon_{si}} (x - x_n), \quad 0 \leq x \leq x_n$$

$$E_{max} = \frac{qN_d}{\epsilon_0 \epsilon_{si}} x_n, \quad E_{max} = \frac{qN_a}{\epsilon_0 \epsilon_{si}} x_p$$

condition for continuity of the field ,

$$N_a x_p = N_d x_n$$

Potential distribution:

$$\phi_j = -\int E(x) dx$$

$$\phi = \frac{qN_a}{\epsilon_0 \epsilon_{si}} x \left(x_p + \frac{x}{2}\right), \quad -x_p \leq x \leq 0$$

$$\phi = \frac{qN_d}{\epsilon_0 \epsilon_{si}} x \left(x_n + \frac{x}{2}\right), \quad 0 \leq x \leq x_n$$

Charge density:

$$\rho = \begin{cases} -qN_a, & -x_p \leq x \leq 0 \\ qN_d, & 0 \leq x \leq x_n \\ 0, & x \leq -x_p \wedge x \geq x_n \end{cases}$$

Depletion layer width:

$$V_a = 0$$

$$x_n = \sqrt{\frac{2\epsilon_{si}\epsilon_0}{q} \frac{N_a}{N_d(N_a + N_d)} \phi_j}$$

$$x_p = \sqrt{\frac{2\epsilon_{si}\epsilon_0}{q} \frac{N_d}{N_a(N_a + N_d)} \phi_j}$$

$$W = x_n + x_p$$

$$W = \sqrt{\frac{2\epsilon_{si}\epsilon_0}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \phi_j}$$

For a one-sided junction

$$W \approx \sqrt{\frac{2\epsilon_{si}\epsilon_0 \phi_j}{qN}}$$

$$V_a \neq 0 \wedge \phi_j - V_a > 0 \rightarrow \phi_j \equiv (\phi_j - V_a)$$

Diode current:

$$I_D = I_S \left[ e^{\frac{V_a}{nV_{th}}} - 1 \right], \quad n = 1?$$

$$I_S = qAn_i^2 \left[ \frac{D_p}{L_p N_d} \coth\left(\frac{W_n}{L_p}\right) + \frac{D_n}{L_n N_a} \coth\left(\frac{W_p}{L_n}\right) \right]$$

$$I_S = qAn_i^2 \left[ \frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a} \right] \quad \text{for } W \gg L$$

For a linearly graded junction

Build in Potential:

$$\phi_i = V_{th} \ln\left(\frac{a^2 W_o^2}{4 n_i^2}\right)$$

Field Distribution:

$$E(x) = E_{max} \left( 1 - \frac{4x^2}{W^2} \right)$$

$$E_{max} = -\frac{qaW^2}{8\epsilon_0\epsilon_{si}}$$

Potential distribution

$$\phi(x) = \frac{qa}{2\epsilon_{si}\epsilon_0} \left( \frac{W^2}{4} x - \frac{x^3}{3} \right)$$

Depletion layer width:

$$W = \left[ \frac{12\epsilon_0\epsilon_{si}}{qa} (\phi_i - V_a) \right]^{\frac{1}{3}}$$

$$W_o = \left[ \frac{12\epsilon_0\epsilon_{si}}{qa} \phi_i \right]^{\frac{1}{3}}$$

**The MOS Capacitor**

Carrier concentration in bulk:

$$p_s = n_i e^{\frac{E_i(\text{surface}) - E_F}{kT}} = n_i e^{\frac{E_F - E_i(\text{bulk})}{kT}}$$

For a p-type s.c.:

$$\phi_F = \frac{kT}{q} \ln\left(\frac{N_a}{n_i}\right)$$

For a n-type s.c.:

$$\phi_F = -\frac{kT}{q} \ln\left(\frac{N_d}{n_i}\right)$$

At depletion-inversion transition point:

$$\phi_{sf} = 2\phi_F$$

Electric field distribution:

For a p-type substrate:

$$E(x) = -\frac{qN_a}{\epsilon_{si}\epsilon_0} (W - x), \quad 0 \leq x \leq W$$

For a n-type substrate:

$$E(x) = \frac{qN_d}{\epsilon_{si}\epsilon_0} (W - x), \quad 0 \leq x \leq W$$

Potential distribution:

For a p-type substrate:

$$\phi(x) = \frac{qN_a}{2\epsilon_{si}\epsilon_0} (W - x)^2, \quad 0 \leq x \leq W$$

For a n-type substrate:

$$\phi(x) = -\frac{qN_d}{2\epsilon_{si}\epsilon_0} (W - x)^2, \quad 0 \leq x \leq W$$

Depletion width:

$$W = \left[ \frac{2\epsilon_{si}\epsilon_0}{qN_a} \phi_{sf} \right]^{\frac{1}{2}}$$

Maximum depletion width:

$$W_T = \left[ \frac{2\epsilon_{si}\epsilon_0}{qN_a} 2\phi_F \right]^{\frac{1}{2}}$$

Capacitance

Oxide capacitance:

$$C_{ox} = \frac{\epsilon_0\epsilon_{ox}A}{t_{ox}}$$

S.c. Capacitance:

$$C_s = \frac{\epsilon_0\epsilon_{si}A}{W}$$

MOS Capacitance:

Accumulation region:

$$C(\text{acc}) \approx C_{ox} = \frac{\epsilon_0\epsilon_{ox}A}{t_{ox}}$$

Depletion region:

$$C(\text{depl}) = \frac{C_{ox} C_s}{C_{ox} + C_s} = \frac{C_{ox}}{1 + \frac{\epsilon_{ox} W}{\epsilon_{si} t_{ox}}}$$

Inversion region:

$$C(\text{inv}) = C_{ox} \text{ for } \omega \rightarrow 0$$

$$C(\text{inv}) = C(\text{depl})_{\min} = \frac{C_{ox} C_s}{C_{ox} + C_s} = \frac{C_{ox}}{1 + \frac{\epsilon_{ox} W_T}{\epsilon_{si} t_{ox}}} \text{ for } \omega \rightarrow \infty$$

Generalized capacitance equation:

$$C = \frac{C_{ox}}{\sqrt{1 + \frac{V_G}{V_\delta}}}$$

$$\text{where } V_\delta = \frac{q \epsilon_{si} t_{ox}^2}{2 \epsilon_{ox}^2 \epsilon_0} N_a \text{ (p-bulk)}$$

$$N_a \rightarrow -N_d \text{ (n-bulk)}$$

Gate voltage relationships:

$$V_G = \phi_{sf} + \frac{\epsilon_{si}}{\epsilon_{ox}} t_{ox} E_s, \quad E_s = E(0), \quad 0 \leq \phi_{sf} \leq 2 \phi_F$$

### The MS-Contact

S.c. work function:

$$q \phi_s = q \chi + (E_C - E_F)_{FB}$$

Barrier width:

MS (n-type) rectifying contact:

$$\phi_B = \phi_M - \chi$$

MS (p-type) rectifying contact:

$$q \phi_B = E_G + q \chi - q \phi_M$$

Build-in potential:

$$V_{bi} = \left[ \phi_B - \frac{(E_C - E_F)_{FB}}{q} \right]$$

Charge distribution:

MS (n-type) rectifying contact:

$$\rho \approx \begin{cases} qN_d, & 0 \leq x \leq W \\ 0, & x > W \end{cases}$$

MS (p-type) rectifying contact:

$$\rho \approx \begin{cases} -qN_a, & 0 \leq x \leq W \\ 0, & x > W \end{cases}$$

Field distribution:

MS (n-type) rectifying contact:

$$E(x) = -\frac{qN_d}{\epsilon_{si} \epsilon_0} (W - x), \quad 0 \leq x \leq W$$

MS (p-type) rectifying contact:

$$E(x) = \frac{qN_a}{\epsilon_{si} \epsilon_0} (W - x), \quad 0 \leq x \leq W$$

Potential distribution:

MS (n-type) rectifying contact:

$$V(x) = -\frac{qN_d}{2 \epsilon_{si} \epsilon_0} (W - x)^2, \quad 0 \leq x \leq W$$

MS (p-type) rectifying contact:

$$V(x) = \frac{qN_a}{2 \epsilon_{si} \epsilon_0} (W - x)^2, \quad 0 \leq x \leq W$$

Depletion layer width:

$$W = \left[ \frac{2 \epsilon_{si} \epsilon_0}{qN_d} (V_{bi} - V_a) \right]^{\frac{1}{2}}$$

### The MOSFET:

Threshold voltage  $V_T$ :

For an ideal n-channel (p-bulk) device:

$$V_T = 2 \phi_F + \frac{\epsilon_{si} t_{ox}}{\epsilon_{ox}} \sqrt{\frac{4 q N_a}{\epsilon_{si} \epsilon_0}} \phi_F$$

For an ideal p-channel (n-bulk) device:

$$V_T = 2 \phi_F - \frac{\epsilon_{si} t_{ox}}{\epsilon_{ox}} \sqrt{\frac{4 q N_d}{\epsilon_{si} \epsilon_0}} (-\phi_F)$$

Effective mobility:

$$\bar{\mu}_n = \frac{\mu_0}{1 + \theta (V_{GS} - V_T)}$$

Relation between  $I_{DS}$  and  $V_{DS}$  from square law model:

$$I_{DS} = \frac{W \bar{\mu}_n C_{ox}}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$\text{for } 0 \leq V_{DS} \leq V_{DSat} \wedge V_{GS} \geq V_T$$

$$I_{DSat} = \frac{W \bar{\mu}_n C_{ox}}{2L} (V_{GS} - V_T)^2$$

$$V_{DSat} = V_{GS} - V_T$$

Relation between  $I_{DS}$  and  $V_{DS}$  from bulk charge theory:

$$I_{DS} = \frac{W \bar{\mu}_n C_{ox}}{L} \left\{ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} - \frac{4}{3} V_W \phi_F \left[ \left( 1 + \frac{V_{DS}}{2 \phi_F} \right)^{\frac{3}{2}} - \left( 1 + \frac{3 V_{DS}}{4 \phi_F} \right) \right] \right\}$$

with

$$V_W = \frac{q N_a W_T}{C_{ox}}, \quad C_{ox} = \frac{\epsilon_{ox} \epsilon_0}{t_{ox}}, \quad W_T = \left[ \frac{2 \epsilon_{si} \epsilon_0}{q N_a} (2 \phi_F) \right]^{\frac{1}{2}}$$

$$V_{DSat} = V_{GS} - V_T - V_W \left\{ \left[ \frac{V_{GS} - V_T}{2\phi_F} + \left(1 + \frac{V_W}{4\phi_F}\right)^2 \right]^{\frac{1}{2}} - \left(1 + \frac{V_W}{4\phi_F}\right) \right\}$$

Cut-off frequency:

$$f_{max} = \frac{g_m}{2\pi C_{ox}} = \frac{\mu_n V_{DS}}{2\pi L^2}$$

### Bipolar Junction Transistor:

Relation between currents

$$I_E = I_B + I_C$$

Relation between equations:

$$V_{EB} + V_{BC} + V_{CE} = 0$$

Emitter efficiency for a pnp BJT:

$$\gamma = \frac{I_{Ep}}{I_E} = \frac{I_{Ep}}{I_{Ep} + I_{En}} = \frac{1}{1 + \left( \frac{D_E L_B N_B \sinh(W/L_B)}{D_B L_E N_E \cosh(W/L_B)} \right)}$$

$$\text{assuming } (W/L_B) \ll 1: \gamma = 1 - \frac{D_E N_B W}{D_B N_E L_E}$$

Base transport factor for a pnp BJT:

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} = \frac{1}{\cosh(W/L_B)} \quad I_{B2} = 0: \rightarrow \alpha_T = 1$$

CB d.c. current gain

$$\alpha_{dc} = \alpha_T \gamma = \frac{1}{\cosh\left(\frac{W}{L_B}\right) + \left(\frac{D_E L_B N_B}{D_B L_E N_E} \sinh\left(\frac{W}{L_B}\right)\right)}$$

CE d.c. current gain:

$$\beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} = \frac{1}{\alpha_{dc} - 1} \quad \beta_{dc} = \frac{D_D N_E L_E}{D_B N_B W}$$

$$= \frac{1}{\cosh\left(\frac{W}{L_B}\right) + \left(\frac{D_E L_B N_B}{D_B L_E N_E} \sinh\left(\frac{W}{L_B}\right)\right) - 1}$$

$$\gg I_{CE0}: \quad \beta_{dc} \approx \frac{I_C}{I_B}$$

Terminal currents:

$$I_E = qA \left[ \left( \frac{D_E}{L_E} n_{E0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) - \left( \frac{D_B}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} \right) (e^{qV_{CB}/kT} - 1) \right]$$

$$I_C = qA \left[ \left( \frac{D_B}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) - \left( \frac{D_C}{L_C} n_{C0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right) (e^{qV_{CB}/kT} - 1) \right]$$

Ebers-Moll model:

$$I_{F0} \equiv qA \left( \frac{D_E}{L_E} n_{E0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right)$$

$$I_{R0} \equiv qA \left( \frac{D_C}{L_C} n_{C0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right)$$

$$I_E = I_{F0} (e^{qV_{EB}/kT} - 1) - \alpha_R I_{R0} (e^{qV_{CB}/kT} - 1)$$

$$I_C = \alpha_F I_{F0} (e^{qV_{EB}/kT} - 1) - I_{R0} (e^{qV_{CB}/kT} - 1)$$

$$\alpha_F I_{F0} = \alpha_R I_{R0} \equiv qA \frac{D_B}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} = I_S$$