Semiconductor Materials: Physics and Technology

Resitivity:

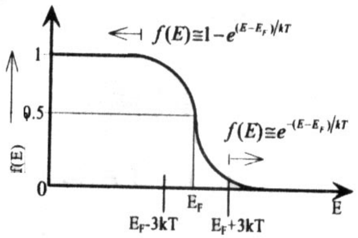
$$\rho=\frac{1}{\sigma}, \quad R=\rho\cdot\frac{L}{Wh}$$

No. of occupied energy levels:

$$n=\int_{E_c}^{E_{top}} N(E)\cdot F(E)\,dE$$

$$p=\int_{-\infty}^{E_v} N(E)\cdot(1-F(E))\,dE$$

$$F(E)=\frac{1}{1+e^{\frac{(E-E_F)}{kT}}}$$



Non degenerate s.c. carrier densities:

$$n=N_c e^{\left(-\frac{E_c-E_F}{kT}\right)}$$

$$p=N_v e^{\left(-\frac{E_F-E_V}{kT}\right)}$$

Intrinsic s.c. carrier density:

$$n=p=n_i(T), \quad n\cdot p=n_i^2$$

$$n_i=\sqrt{N_c N_v}\,e^{\left(\frac{-E_g}{2kT}\right)}$$

$$n_i^2=15{,}05\cdot 10^{32}\cdot T^3\cdot e^{(-14000/T)}$$

$$N_{c,v}\equiv 2\left(\frac{2\pi m_{n,p}^{\circ} kT}{h^2}\right)^{\frac{3}{2}}, \quad h=2\pi\hbar$$

$$n=N_d, \quad p=\frac{n_i^2}{N_d}; \quad N_d\gg N_a, \quad N_d\gg n_i \quad n-type$$

$$p=N_a, \quad n=\frac{n_i^2}{N_a}; \quad N_a\gg N_d, \quad N_a\gg n_i \quad p-type$$

Neutralitätsbedingung im dot. s.c.:

$$n-p=N_d \qquad n-type$$

$$p-n=N_a \qquad p-type$$

Carrier densities in a doped s.c.:

$$n=\frac{N_d-N_a}{2}+\sqrt{\left(\frac{N_d-N_a}{2}\right)^2+n_i^2}$$

$$p=\frac{N_a-N_d}{2}+\sqrt{\left(\frac{N_a-N_d}{2}\right)^2+n_i^2}$$

Fermilevel of an intrinsic s.c.:

$$E_i=\frac{E_c-E_v}{2}+\frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right)=\frac{E_c-E_v}{2}+\frac{3}{4}kT\ln\left(\frac{m_p^{\circ}}{m_n^{\circ}}\right)$$

Fermilevel in a doped s.c.:

$$E_F-E_i=kT\ln\left(\frac{n}{n_i}\right)=-kT\ln\left(\frac{p}{n_i}\right)$$

Drift velocity:

$$\vec{v}_d=\mu\vec{E}$$

Drift current equations:

$$J=\frac{I}{A}$$

$$\vec{J}_{n,drift}=-qn\vec{v}_d=qn\mu_n\vec{E}$$

$$\vec{J}_{p,drift}=qp\vec{v}_d=qp\mu_p\vec{E}$$

Einstein's relations:

$$D_n=\left(\frac{kT}{q}\right)\mu_n, \quad D_p=\left(\frac{kT}{q}\right)\mu_p$$

Resitivity of a s.c.:

$$\rho=\frac{1}{\sigma}=\frac{1}{q(n\mu_n+p\mu_p)}$$

Conductivity of a s.c.:

$$\sigma=\sigma_n+\sigma_p=q(n\mu_n+p\mu_p)$$

Diffusion lengths:

$$L_n=\sqrt{D_n\tau_n}, \quad L_p=\sqrt{D_p\tau_p}$$

Diffusion currents:

$$J_{n,diff}=qD_n\frac{dn}{dx} \quad J_{p,diff}=-qD_p\frac{dp}{dx}$$

Current equations:

$$J_n=J_{n,drift}+J_{n,diff}=qn\mu_n E+qD_n\frac{dn}{dx}$$

$$J_p=J_{p,drift}+J_{p,diff}=qp\mu_p E-qD_p\frac{dp}{dx}$$

Continuity equations:

$$\frac{\partial n}{\partial t}=\frac{1}{q}\frac{\partial J_n}{\partial x}-R_n+G_n$$

$$\frac{\partial p}{\partial t}=-\frac{1}{q}\frac{\partial J_p}{\partial x}-R_p+G_p$$

The pn-Junction

<u>For an aprupt junction:</u>

Build in Potential:

$$\phi_j=V_{th}\ln\left(\frac{N_a N_d}{n_i^2}\right), \quad V_{th}=\frac{kT}{q}$$

Field Distribution:

$$E(x)=\frac{1}{\epsilon_0\epsilon_{si}}\int\rho(x)\,dx$$

$$E(x)=-\frac{qN_a}{\epsilon_0\epsilon_{si}}(x_p+x), -x_p\le x\le 0$$

$$E(x)=-\frac{qN_d}{\epsilon_0\epsilon_{si}}(x-x_n), 0\le x\le x_n$$

$$E_{max}=\frac{qN_d}{\epsilon_0\epsilon_{si}}x_n, \quad E_{max}=\frac{qN_a}{\epsilon_0\epsilon_{si}}x_p$$

condition for continuity of the field ,

$$N_a x_p=N_d x_n$$

Potential distribution:

$$\phi_j=-\int E(x)\,dx$$

$$\phi=\frac{qN_a}{\epsilon_0\epsilon_{si}}x\left(x_p+\frac{x}{2}\right), -x_p\le x\le 0$$

$$\phi=\frac{qN_d}{\epsilon_0\epsilon_{si}}x\left(x_n+\frac{x}{2}\right), 0\le x\le x_n$$

Charge density:

$$\rho=\begin{cases}-qN_a, -x_p\le x\le 0\\ qN_d, 0\le x\le x_n\\ 0, x\le -x_p\wedge x\ge x_n\end{cases}$$

Depletionlayer width:

$$\boldsymbol{V_a=0}$$

$$x_n=\sqrt{\frac{2\epsilon_{si}\epsilon_0}{q}\frac{N_a}{N_d(N_a+N_d)}\phi_j}$$

$$x_p=\sqrt{\frac{2\epsilon_{si}\epsilon_0}{q}\frac{N_d}{N_a(N_a+N_d)}\phi_j}$$

$$W=x_n+x_p$$

$$W=\sqrt{\frac{2\epsilon_{si}\epsilon_0}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\phi_j}$$

$$For\ a\ one-sided\ junction$$

$$W\approx\sqrt{\frac{2\epsilon_{si}\epsilon_0\phi_j}{qN}}$$

$$\boldsymbol{V_a\neq 0}\wedge\phi_j-V_a>0\rightarrow\phi_j\equiv(\phi_j-V_a)$$

Diode current:

$$I_D=I_S\left[e^{\frac{V_a}{nV_{th}}}-1\right],\quad n=1?$$

$$I_S=qAn_i^2\left[\frac{D_p}{L_pN_d}\coth\left(\frac{W_n}{L_p}\right)+\frac{D_n}{L_nN_a}\coth\left(\frac{W_p}{L_n}\right)\right]$$

$$I_S=qAn_i^2\left[\frac{D_p}{L_pN_d}+\frac{D_n}{L_nN_a}\right]\quad for\quad W\gg L$$

For a linearly graded junction

Build in Potential:

$$\phi_i=V_{th}\ln\left(\frac{a^2W_o^2}{4n_i^2}\right)$$

Field Distribution:

$$E(x)=E_{max}\left(1-\frac{4x^2}{W^2}\right)$$

$$E_{max}=-\frac{qaW^2}{8\epsilon_0\epsilon_{si}}$$

Potential distribution

$$\phi(x)=\frac{qa}{2\epsilon_{si}\epsilon_0}\left(\frac{W^2}{4}x-\frac{x^3}{3}\right)$$

Depletion layer width:

$$W=\left[\frac{12\epsilon_0\epsilon_{si}}{qa}(\phi_i-V_a)\right]^{\frac{1}{3}}$$

$$W_o=\left[\frac{12\epsilon_0\epsilon_{si}}{qa}\phi_i\right]^{\frac{1}{3}}$$

The MOS Capacitor

Carrier concentration in bulk:

$$p_s=n_ie^{\frac{E_i(surface)-E_F}{kT}}=n_ie^{\frac{E_F-E_i(bulk)}{kT}}$$

For a p-type s.c.:

$$\phi_F=\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right)$$

For a n-type s.c.:

$$\phi_F=-\frac{kT}{q}\ln\left(\frac{N_d}{n_i}\right)$$

At depletion-inversion transition point:

$$\phi_{sf}=2\phi_F$$

Electric field distribution:

For a p-type substate:

$$E(x)=-\frac{qN_a}{\epsilon_{si}\epsilon_0}(W-x),\quad 0\le x\le W$$

For a n-type substate:

$$E(x)=\frac{qN_d}{\epsilon_{si}\epsilon_0}(W-x),\quad 0\le x\le W$$

Potential distribution:

For a p-type substrate:

$$\phi(x)=\frac{qN_a}{2\epsilon_{si}\epsilon_0}(W-x)^2,\quad 0\le x\le W$$

For a n-type substrate:

$$\phi(x)=-\frac{qN_d}{2\epsilon_{si}\epsilon_0}(W-x)^2,\quad 0\le x\le W$$

Depletion width:

$$W=\left[\frac{2\epsilon_{si}\epsilon_0}{qN_a}\phi_{sf}\right]^{\frac{1}{2}}$$

Maximum depletion width:

$$W_T=\left[\frac{2\epsilon_{si}\epsilon_0}{qN_a}2\phi_F\right]^{\frac{1}{2}}$$

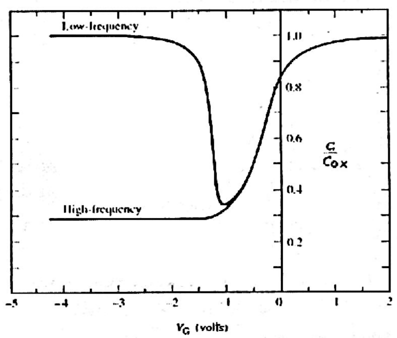
Capacitance

Oxide capacitance:

$$C_{ox}=\frac{\epsilon_0\epsilon_{ox}A}{t_{ox}}$$

S.c. Capacitance:

$$C_s=\frac{\epsilon_0\epsilon_{si}A}{W}$$



MOS Capacitance:

Accumulation region:

$$C(acc)\approx C_{ox}=\frac{\epsilon_0\epsilon_{ox}A}{t_{ox}}$$

Depletion region:

$$C(depl)=\frac{C_{ox}C_s}{C_{ox}+C_s}=\frac{C_{ox}}{1+\frac{\epsilon_{ox}W}{\epsilon_{si}t_{ox}}}$$

Inversion region:

$$C(inv)=C_{ox}\quad for\,\omega\rightarrow 0$$

$$C(inv)=C(depl)_{min}=\frac{C_{ox}C_s}{C_{ox}+C_s}$$

$$=\frac{C_{ox}}{1+\frac{\epsilon_{ox}W_T}{\epsilon_{si}t_{ox}}}\;for\,\omega\rightarrow\infty$$

Generalized capacitance equation:

$$C=\frac{C_{ox}}{\sqrt{1+\frac{V_G}{V_\delta}}}$$

$$where\,V_\delta=\frac{q\epsilon_{si}t_{ox}^2}{2\epsilon_{ox}^2\epsilon_0}N_a\quad(p-bulk)$$

$$N_a\rightarrow -N_d\quad(n-bulk)$$

Gate voltage relationships:

$$V_G=\phi_{sf}+\frac{\epsilon_{si}}{\epsilon_{ox}}t_{ox}E_s,\quad E_s=E(0),\;0\leq\phi_{sf}\leq 2\phi_F$$

The MS-Contact

S.c. work function:

$$q\phi_s=q\chi+(E_C-E_F)_{FB}$$

Barrier width:

MS (n-type) rectifying contact:

$$\phi_B=\phi_M-\chi$$

MS (p-type) rectifying contact:

$$q\phi_B=E_G+q\chi-q\phi_M$$

Build-in potential:

$$V_{bi}=\left[\phi_B-\frac{(E_C-E_F)_{FB}}{q}\right]$$

Charge distribution:

MS (n-type) rectifying contact:

$$\rho\simeq\begin{cases}qN_d, & 0\leq x\leq W\\ 0, & x>W\end{cases}$$

MS (p-type) rectifying contact:

$$\rho\simeq\begin{cases}-qN_a, & 0\leq x\leq W\\ 0, & x>W\end{cases}$$

Field distribution:

MS (n-type) rectifying contact:

$$E(x)=-\frac{qN_d}{\epsilon_{si}\epsilon_0}(W-x),\quad 0\leq x\leq W$$

MS (p-type) rectifying contact:

$$E(x)=\frac{qN_a}{\epsilon_{si}\epsilon_0}(W-x),\quad 0\leq x\leq W$$

Potential distribution:

MS (n-type) rectifying contact:

$$V(x)=-\frac{qN_d}{2\epsilon_{si}\epsilon_0}(W-x)^2,\quad 0\leq x\leq W$$

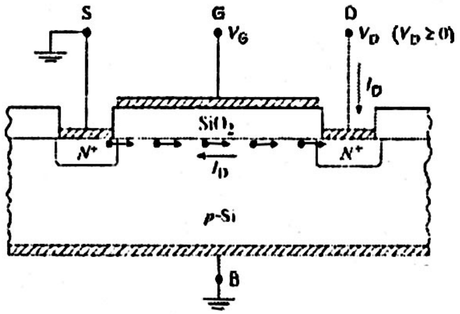
MS (p-type) rectifying contact:

$$V(x)=\frac{qN_a}{2\epsilon_{si}\epsilon_0}(W-x)^2,\quad 0\leq x\leq W$$

Depletion layer width:

$$W=\left[\frac{2\epsilon_{si}\epsilon_0}{qN_d}(V_{bi}-V_a)\right]^{\frac{1}{2}}$$

The MOSFET:



Threshold voltage V_T:

For an ideal n-channel (p-bulk) device:

$$V_T=2\phi_F+\frac{\epsilon_{si}t_{ox}}{\epsilon_{ox}}\sqrt{\frac{4qN_a}{\epsilon_{si}\epsilon_0}\phi_F}$$

For an ideal p-channel (n-bulk) device:

$$V_T=2\phi_F-\frac{\epsilon_{si}t_{ox}}{\epsilon_{ox}}\sqrt{\frac{4qN_d}{\epsilon_{si}\epsilon_0}(-\phi_F)}$$

Effective mobility:

$$\bar{\mu}_n=\frac{\mu_0}{1+\theta(V_{GS}-V_T)}$$

Relation between I_{DS} and V_{DS} from square law model:

$$I_{DS}=\frac{W\bar{\mu}_nC_{ox}}{L}\left[(V_{GS}-V_T)V_{DS}-\frac{V_{DS}^2}{2}\right]$$

$$for\;0\leq V_{DS}\leq V_{DSat}\;\wedge\;V_{GS}\geq V_T$$

$$I_{DSat}=\frac{W\bar{\mu}_nC_{ox}}{2L}(V_{GS}-V_T)^2$$

$$V_{DSat}=V_{GS}-V_T$$

Relation between I_{DS} and V_{DS} from bulk charge theory:

$$I_{DS}=\frac{W\bar{\mu}_n C_{ox}}{L}\left\{(V_{GS}-V_T)V_{DS}-\frac{V_{DS}^2}{2}\right.$$
$$\left.-\frac{4}{3}V_W\phi_F\left[\left(1+\frac{V_{DS}}{2\phi_F}\right)^{\frac{3}{2}}-\left(1+\frac{3V_{DS}}{4\phi_F}\right)\right]\right\}$$

with

$$V_W=\frac{qN_a W_T}{C_{ox}},\quad C_{ox}=\frac{\epsilon_{ox}\epsilon_0}{t_{ox}},\quad W_T=\left[\frac{2\epsilon_{si}\epsilon_0}{qN_a}(2\phi_F)\right]^{\frac{1}{2}}$$

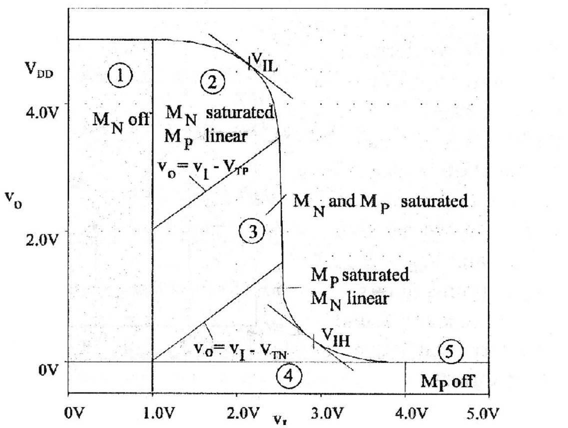
$$V_{DSat}=V_{GS}-V_T-V_W\left\{\left[\frac{V_{GS}-V_T}{2\phi_F}+\left(1+\frac{V_W}{4\phi_F}\right)^2\right]^{\frac{1}{2}}-\left(1+\frac{V_W}{4\phi_F}\right)\right\}$$

Cut-off frequency:

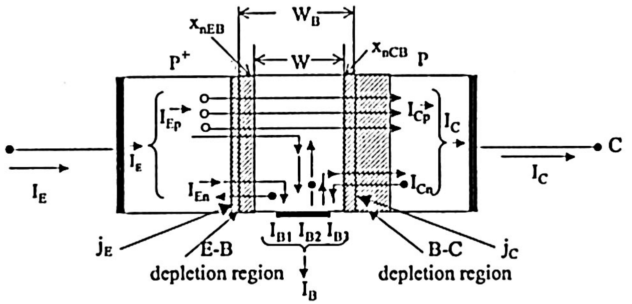
$$f_{max}=\frac{g_m}{2\pi C_{ox}}=\frac{\mu_n V_{DS}}{2\pi L^2}$$

CMOS: Digital Applications

CMOS voltage transfer characteristics of symmetrical inverter



Bipolar Junction Transistor:



Relation between currents

$$I_E=I_B+I_C$$

Relation between equations:

$$V_{EB}+V_{BC}+V_{CE}=0$$

Emitter efficiency for a pnp BJT:

$$\gamma=\frac{I_{Ep}}{I_E}=\frac{I_{Ep}}{I_{Ep}+I_{En}}=\frac{1}{1+\left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\frac{\sinh(W/L_B)}{\cosh(W/L_B)}\right)}$$

$$assuming\ (W/L_B)\ll 1:\quad \gamma=1-\frac{D_E N_B W}{D_B N_E L_E}$$

Base transport factor for a pnp BJT:

$$\alpha_T=\frac{I_{Cp}}{I_{Ep}}=\frac{1}{\cosh(W/L_B)}\qquad I_{B2}=0:\rightarrow\alpha_T=1$$

CB d.c. current gain

$$\alpha_{dc}=\alpha_T\gamma=\frac{1}{\cosh\left(\frac{W}{L_B}\right)+\left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\sinh\left(\frac{W}{L_B}\right)\right)}$$

CE d.c. current gain:

$$\beta_{dc}=\frac{\alpha_{dc}}{1-\alpha_{dc}}=\frac{1}{\frac{1}{\alpha_{dc}-1}}\qquad \beta_{dc}=\frac{D_D N_E L_E}{D_B N_B W}$$

$$=\frac{1}{\cosh\left(\frac{W}{L_B}\right)+\left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\sinh\left(\frac{W}{L_B}\right)\right)-1}$$

$$\gg I_{CE0}:\qquad \beta_{dc}\simeq\frac{I_C}{I_B}$$

Terminal currents:

$$I_E=qA\left[\left(\frac{D_E}{L_E}n_{E0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)\left(e^{qV_{EB}/kT}-1\right)-\left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)\left(e^{qV_{CB}/kT}-1\right)\right]$$

$$I_C=qA\left[\left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)\left(e^{qV_{EB}/kT}-1\right)-\left(\frac{D_C}{L_C}n_{C0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)\left(e^{qV_{CB}/kT}-1\right)\right]$$

Ebers-Moll model:

$$I_{F0}\equiv qA\left(\frac{D_E}{L_E}n_{E0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)$$

$$I_{R0}\equiv qA\left(\frac{D_C}{L_C}n_{C0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)$$

$$I_E=I_{F0}\left(e^{qV_{EB}/kT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{CB}/kT}-1\right)$$

$$I_C=\alpha_F I_{F0}\left(e^{qV_{EB}/kT}-1\right)-I_{R0}\left(e^{qV_{CB}/kT}-1\right)$$

$$\alpha_F I_{F0}=\alpha_R I_{R0}\equiv qA\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}=I_S$$